Automation and Cross-Occupation Spillovers

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Job Market Paper
Latest version

Abstract

This paper demonstrates how, through the capital reallocation channel, increased automation in routine occupations has reduced employment and wages in non-routine occupations. Automation in routine occupations absorbs capital from non-routine occupations, reducing employment and wages in the latter. This mechanism is referred to as automation cross-occupation spillovers. Between 1980 and 2010, automation reduced average labor income by 27%. Cross-occupation spillover is responsible for 62% of this drop. For example, the increase in automation in the 10% most routine-intensive occupations between 1980 and 2010 reduced average labor income in the 90% least routine-intensive occupations by 2.04%. Furthermore, I find that automation has contributed to the rise of inequality in the United States. Indeed, automation accounts for 30.3% of the increase in occupational labor income inequality between 1980 and 2010.

Keywords: Automation, cross-occupation Spillovers, Labor Income

(JEL E22, E23, E24, O33, O41)

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[0] I would like to thank my supervisors, Bariş Kaymak and Joao Alfredo Galindo da Fonseca, for their advice and support. I would like to thank also the professors at the Université de Montréal and CEA conference attendees for their insightful comments.

November 29, 2022

Introduction

In this paper, I study the impact of automation on the distribution of wages and employment across occupations. I demonstrate, in particular, how automation of tasks in an occupation affects employment and wages, not only in that occupation, but also in other occupations via reallocation of capital investment across occupations.

To that end, I develop a multi-occupation model of automation and characterize the optimal allocation of capital across occupations. I then derive analytical expressions for the marginal impact of automation on the equilibrium distribution of wages and employment across occupations. For each occupation, these expressions distinguish between the direct effects of automation of tasks performed in that occupation from the spillovers effects that arise in equilibrium due to automation of tasks performed in other occupations. Using comparative statics, I fully characterize automation's short-run and long-run effects as well as the transitional dynamics after an increase in the automation of production.

In the short-run, task automation reduces wages and raises the productivity of capital in the automated occupation. Because aggregate capital is fixed in the short-run, automation draws capital from other occupations into the automated occupation. As a result, labor demand falls in the latter reducing wages and employment. The reallocation of capital towards the automated occupation mitigates the negative effects of automation on labor demand in that occupation. The net effect is negative because capital inflows are insufficient to raise labor demand. All else being equal, the direct effect of automation is greater than the spillover effect, implying that routine occupations are the most affected by automation. The magnitude of the spillover effect is proportional to the size of the automated occupation. Indeed, large occupations absorb an exorbitant amount of capital.

In the long-run, automation encourages capital accumulation, which raises labor demand in all occupations over time. In the new steady state, the level of employment and wages in the automated occupation and the other occupations are higher than in the short run.

During the transition, the capital stock increases. The main driver of the increase in the capital stock is a temporary increase of the interest rate. Although the interest rate at the two steady states is equal to the discount rate, it rises sharply in the short run and then falls back to the level of the discount rate during the transition. The rise in the interest rate is driven by the fact that automation increases capital productivity, implying an increase in capital demand. The increase in the interest rate encourages households to increase their savings. The dispersion of wages and employment increases along the transition. Indeed, employment and wages rise faster in occupations with relatively high labor productivity.

In Section 3, I provide a quantitative assessment of the effect of automation on wages and employment across occupations. I begin by calibrating the model to the United States in 2010 using data from the America Community Survey. For each occupation, I define the potential automation risk by that occupation's content of routine tasks. Assuming that actual level of automation in each occupation is proportional to its potential level, I determine the level of automation by calibrating it to match the aggregate labor share in 2010 in the US. In the benchmark economy, the level of employment and wages in an occupation declines with its routine share as in the data.

Next, I simulate the 1980s US economy, by reducing the level of automation so that the aggregate labor share matches its value in 1980. Comparing the simulated equilibria in 1980 and 2010, I find that automation has reduced average labor income by 27%. Spillovers effects of automation across occupations is responsible for 62% of this drop. For illustration, between 1980 and 2010, the increase in automation in the ten percent most routine-intensive occupations reduced average labor income in the 90% least routine-intensive occupations by 2.04%.

I find that automation has contributed to the rise of inequality in the United States. Indeed, between 1980 and 2010, automation contributed by 30.3% to the increase in occupational labor income inequality. Although the spillovers exacerbate the fall in average labor income caused by automation, they lessen the impact of automation on inequality. In other words, if spillovers had not occurred, the increase in inequality would have been greater.

According to conventional wisdom, some workers (i.e those operating in routine occupations) are highly vulnerable to the consequences of new technologies, while others are relatively unscathed. In this regard, Frey and Osborne (2013) and Manyika (2017) estimate that 47 to 50% of US workers are at risk of automation. However, once spillovers are taken into account, one can see that the entire workforce is vulnerable. Especially, increased automation in routine occupations reduces employment and wages in manual and abstract occupations.

Cross-occupation spillover is a novel effect of automation on the labor market that the paper adds to the literature. The automation effects known in the literature, are the productivity effect and the displacement effect. The former stems from the fact that automation lowers the cost, which raises the level of production and, as a result, the factors demand (Acemoglu and Restrepo (2018a)). The displacement effect is caused by the fact that new machines will replace workers in certain tasks (Acemoglu and Restrepo (2021)).

There has been discussion about how automation will affect employment, wages, and overall stability. So far, the literature has reached opposing conclusions. On the one hand, Acemoglu et al. (2020), Acemoglu and Restrepo (2019), Acemoglu and Restrepo (2018b), and Bessen et al. (2019) conclude that automation has a negative impact. On the other hand, Autor (2015), Graetz and Michaels (2018) conclude that automation has a positive impact. This paper shows how automation causes negative spillovers through the capital reallocation mechanism. This is, to the best of my knowledge, the first paper that addresses automation cross-occupation spillovers.

The paper is also related to the wage polarization literature, which emphasizes the disproportionate effect of technical change among workers Autor (2014). Autor (2015) and Autor and Dorn (2013) show that routine occupation workers have been hit the hardest by increased automation. In this paper, I show that, while routine occupations are the hardest hit by automation, non-routine occupation workers are not immune to automation because of spillovers. Without cross-occupation spillovers, wage polarization would have been even more pronounced.

The paper is divided into four sections. The first section describes the multi-occupation model of automation. The second section solves the model equilibrium, presents an analysis of comparative statics with respect to automation in the short and the long run, and characterizes the transitions. The third section calibrates the model parameters to the US economy, and quantifies the effect of cross-occupation spillovers on the change in employment and wages attributable to automation. Section four concludes.

1. A Multi-Occupation Model of Automation

Occupations are diverse in terms of their automatable task content. Some occupations, are routine task intensive and thus highly automatable, whereas others are not. To account for this heterogeneity, I extend the task-based model of automation presented in Acemoglu and Restrepo (2018b), Zeira (1998), Acemoglu and Autor (2011) to multiple occupations. The model is in continuous time, with heterogeneous households and a representative producer who manufactures the final good using the services of all occupations.

1.1. Preferences

There are n types of households in the economy, each type is endowed with labor skills that are specific to an occupation. The representative household of type i, $i \in \{1, 2, ..., n\}$, denoted by H_i, only provides labor to occupation i. The household H_i lifetime utility function is given by:

$$\int_0^{\infty} \mathrm{e}^{-\rho t} \frac{\left(c_i\left(t\right) \mathrm{e}^{-\tau l_i(t)}\right)^{1-\theta}}{1-\theta} \mathrm{d}t \tag{1}$$

where $c_i(t)$, and $l_i(t)$ denote the household's consumption and labor supply, and ρ denotes the discount rate between 0 and 1. θ is the inverse of the intertemporal elasticity of substitution. The utility function's concavity requires that $\theta > 1$. $\tau > 0$ represents the distaste for work[1].

[1] According to the hedonic wage theory, some occupations are more difficult to perform than others, so leisure tastes should be heterogeneous, in the sense that it is higher for difficult occupations than for others. (see Cahuc et al. (2014))

I assume that the marginal disutilities of work are the same across occupations.

The household maximizes her lifetime utility function, by selecting her consumption and labor for each period of time subject to the following budget constraint.

$$\dot{a}_i(t) = w_i(t)\, l_i(t) + R(t)\, a_i(t) - c_i(t), \tag{2}$$

where $a_i(t)$ denotes the household wealth at time t, and is positive; w_i is the wage rate in occupation i, and $R(t)$ is the interest rate on capital. The formal problem of the households is the following.

$$\max_{\{c_i(t),a_i(t),l_i(t)\}_{t\geq 0}} \int_0^\infty \mathrm{e}^{-\rho t} \frac{\left(c_i(t)\, \mathrm{e}^{-\tau l_i(t)}\right)^{1-\theta}}{1-\theta} \mathrm{d}t$$

s.t.

$$\dot{a}_i(t) = w_i(t)\, l_i(t) + R(t)\, a_i(t) - c_i(t)$$

1.2. Production

In the economy, there are n occupations that provide intermediary services. The production function of the gross output is a Cobb-Douglas aggregate of the output produced by the occupations. This implies that the elasticity of substitution between occupations is unity. Appendix D discusses the implications of a CES production function.

$$Y(t) = \prod_{i=1}^{n} y_i(t)^{\alpha_i} \tag{3}$$

$Y(t)$ is the gross output, and $y_i(t)$ denotes the output of occupation i. n occupations generate gross output, each with a share α_i, and $\sum_{i=1}^{n} \alpha_i = 1$. Unlike industries, the outputs of occupations are intermediate goods rather than final goods.

The production within an occupation $y_i(t)$ is carried out through a series of tasks x with measure one. Occupations necessitate the use of two types of factors: automation capital and labor. The automation capital operates in routine tasks that engineers are capable of automating, while labor operates in the remaining tasks. Let's denote by η_i the proportion of automated tasks. Task x's output is formalized as follow:

$$y_{ix}(t) = \begin{cases} \gamma_i l_{ix}(t) + k_{ix}(t), & \text{if } x \in [0, \eta_i] \\ \gamma_i l_{ix}(t), & \text{if } x \in (\eta_i, 1] \end{cases} \tag{4}$$

where $k_{ix}(t)$, and $l_{ix}(t)$ are respectively the automation capital, and labor used in the task x. γ_i denotes the productivity of labor in occupation i and y_{ix} is the output of task x of occupation i.

Automation capital is homogeneous and can be transferred between occupations. Several examples show that capital mobility exists across occupations. First, the centralised financial market provides funds for the purchase of automating machines. These funds are divided among various occupations. When automation occurs, redistribution is readjusted. Second, electronic devices such as computers, telephones, and robots can be converted to perform different types of task. This assumption is the mainstay of the model. Labor, on the other hand, is occupation specific.

The producer minimizes the total production costs conditional on the level of output $Y(t)$. More formally

$$\min_{\{l_i(t),k_i(t)\}_{i=1}^n} \sum_{i=1}^n w_i(t)l_i(t) + R(t)k_i(t)$$

s.t.

$$\prod_{i=1}^n \left(\int y_{ix}(t)\mathrm{d}x \right)^{\alpha_i} \geq Y(t)$$

where $l_i(t) = \int l_{ix}(t)\mathrm{d}x$ and $k_i = \int k_{ix}(t)\mathrm{d}x$.

This problem can be divided into two simpler subproblems. The first is to calculate the prices p_i for the occupations, which are defined as the lowest cost of producing one unit of output of occupation i.

$$p_i(t) = \min_{l_i(t),k_i(t)} \left\{ w_i(t)l_i(t) + R(t)k_i(t) \text{ such that } \int y_{ix}(t)\mathrm{d}x = 1 \right\}$$

The second subproblem is to determine the optimal occupational output in order to minimize the total cost.

$$\{y_i(t)\}_{i=1}^n = \underset{y_1(t),\ldots,y_n(t)}{\operatorname{argmin}} \left\{ \sum_{i=1}^n p_i(t)y_i(t) \text{ such that } \prod_{i=1}^n y_i(t)^{\alpha_i} = Y(t) \right\}$$

1.3. Equilibrium

Let $\{\eta_i\}, i \in \{1, 2, ..., n\}$ be the distribution of automation, and $K(0)$ the initial capital stock. An equilibrium is a set of factor prices $\{w_i(t), R(t)\}, i \in \{1, 2, ..., n\}$, the aggregate output $Y(t)$, a stock of capital $K(t)$ s.t.

(i) For all i, $k_i(t)$, and $l_i(t)$ are allocated in a cost minimizing way to produce $y_i(t)$ given the factor prices. The occupation i price $p_i(t)$ denotes the minimum cost of producing one unit of $y_i(t)$.

(ii) $\{y_i(t)\}, i \in \{1, 2, ..., n\}$ are allocated in a cost minimizing way to produce $Y(t)$, given occupation prices $\{p_i(t)\}, i \in \{1, 2, ..., n\}$.

(iii) For all i, $a_i(t)$, $c_i(t)$ and $l_i(t)$ maximize household H_i utility, given $w_i(t)$ and $R(t)$.

(iv) Capital market clears

$$\sum_{i=1}^n \zeta_i a_i(t) = K(t) = \sum_{i=1}^N k_i(t) \tag{5}$$

where ζ_i is the weight of H_i in the economy.

The model provides the distribution of automation and the initial stock of capital exogenously. The condition in equation (5) represents the aggregate capital market clearing condition. Because capital is homogeneous and flows across occupations, the capital market clears at the aggregate level. The aggregate capital supply is represented on the left hand side of equation (5). It is the weighted sum of each representative household's savings. Indeed, H_i represents the workers in occupation i, and the number of workers varies by occupation.

Because labor is occupation specific, the labor market clears at the occupation level. It is represented by l_i, the equilibrium employment that solves both the household and the producer problems.

A steady state equilibrium is one in which factor quantities and prices remain constant over time.

2. Model Analysis

In this section, I derive the equilibrium quantities and prices, characterize the steady state equilibrium, and compute the comparative statics with respect to automation in the short and long run.

2.1. Factors Behavior

Given the definition of the production function in the specific task of an occupation, as stated in equation (4), the production of intermediate goods y_i has the following Cobb Douglas form.

$$y_i(t) = \left(\frac{k_i(t)}{\eta_i}\right)^{\eta_i} \left(\frac{\gamma_i l_i(t)}{1-\eta_i}\right)^{1-\eta_i} \tag{6}$$

Proof 1. *See Acemoglu and Restrepo (2018b)*

Automation boosts capital productivity while decreasing labor share. Factors demands in occupation i are given by

$$k_i(t) = \eta_i \left(\frac{w_i(t)}{R(t)}\right)^{1-\eta_i} y_i(t)$$

$$l_i(t) = (1 - \eta_i) \left(\frac{R(t)}{w_i(t)} \right)^{\eta_i} y_i(t)$$

When the cost of automation capital falls or the cost of labor rises, the demand for automation capital rises. It may also rise as a result of an increase in gross output. $p_i(t)$, as defined above has the following expression:

$$p_i(t) = R(t)^{\eta_i} w_i(t)^{1-\eta_i}$$

Given the distribution of occupation prices, the optimal demand of intermediate good $y_i(t)$ for a given level of aggregate production $Y(t)$ is

$$y_i(t) = \prod_{k=1}^{n} \left(\frac{\alpha_i}{\alpha_k} \frac{p_k(t)}{p_i(t)} \right)^{\alpha_k} Y(t)$$

The demand for factors are used to determine capital allocation across occupations, for a given stock of capital.

The capital is more productive in occupations where the proportion of automated tasks is higher. However, as more capital is allocated, productivity falls. As a result, once the aggregate stock is determined, the producer has only one scheme for distributing capital across occupations. Proposition 1 presents that capital allocation strategy.

Proposition 1. *Let $K(t)$ denote the total stock of automating capital available to the producer. The optimal allocation of capital is such that*

$$k_i(t) = \frac{\alpha_i \eta_i}{\bar{\eta}} K(t) \tag{7}$$

where $\bar{\eta} = \sum_{i=1}^{n} \alpha_i \eta_i$, is the aggregate automation.

Proof 2. *See Appendix A.1*

Occupations receive a fraction of the total capital according to the factor shares within the occupations, and the occupations shares in the aggregate output. The capital allotted is proportional to the proportion of automated tasks. This analysis supports Autor and

Dorn (2013) finding that routine-intensive occupations use more computers than any other occupation. Proposition 1 sheds light on the source of the spillovers, namely the capital reallocation. Increased automation in one occupation reduces capital available for other occupations, the labor demand declines in the latter because the capital is scarcer. One corollary of the latter proposition is that the aggregate production is a functional of the stock of automating capital.

Corollary 1.

$$Y(t) = AK(t)^{\bar{\eta}} \prod_{i=1}^{n} l_i(t)^{\alpha_i(1-\eta_i)} \tag{8}$$

where $A = \frac{\prod_{i=1}^{n} \left(\frac{\alpha_i^{\eta_i} \gamma_i^{1-\eta_i}}{(1-\eta_i)^{1-\eta_i}} \right)^{\alpha_i}}{\bar{\eta}^{\bar{\eta}}}$ *is the aggregate TFP*

Thus, the framework features a standard Cobb-Douglas production function, with the capital share equals to the aggregate automation. Corollary 1 implies that automation reduces labor share.

Next, I investigate the existence and uniqueness of the steady state equilibrium. I characterize the steady state by equations that give the factor prices and the capital stock. Let $\Lambda_0(.)$ be the Lambert function, also known as the omega function or product logarithm[2]. Proposition 2 states the existence and uniqueness of the steady state.

Proposition 2. *The steady state equilibrium exists, it is unique, and characterized by the following equations*

$$R = \rho \tag{9}$$

$$\sum_i \frac{\zeta_i \phi g_i}{\Lambda_0 (\phi g_i K)} = \rho + \sum_i \zeta_i g_i \tag{10}$$

$$w_i = \frac{\phi \tau g_i K}{\Lambda_0 (\phi g_i K)} \tag{11}$$

$$l_i = \frac{\Lambda_0 (\phi g_i K)}{\phi \tau} \tag{12}$$

where $g_i = \frac{\alpha_i(1-\eta_i)}{\bar{\eta}} R$ *and* $\phi = \frac{\theta - 1}{\theta}$

[2] The Lambert function is commonly denoted as W_0. To prevent any confusion with the wage, I name it otherwise. The Lambert function is the inverse function of xe^x.

Proof 3. *see Appendix B*

Equations (9) to (12) characterize the quantities and prices of factors at the steady state. Equation (9) gives the level of the interest rate. The interest rate must equal the discount rate to rule out the intertemporal labor supply tradeoff. The capital function is represented by equation (10). This equation admits a single solution in K, which is the steady-state level of the capital stock. Equations (11) and (12) are respectively the level of wages and employment as a function of the stock of capital.

2.2. Effects of Automation

The following exercise consists in disentangling the short and long run spillovers of automation by decomposing wages and employment into their respective components. The short-term automation effect is caused by increased automation while capital remains constant. The capital stock is completely flexible in the long run.

2.2.1. Short Run Effect of Automation

The following proposition computes the short-term automation effects in a more comprehensive manner, regardless of household preferences.

Proposition 3. *Let ϵ_i denote the elasticity of labor supply in occupation i. Then*

$$\frac{\partial \log w_i}{\partial \eta_i} = -\frac{1}{1+\epsilon_i}\left(\frac{\alpha_i}{\bar{\eta}} + \frac{1}{1-\eta_i}\right) \tag{13}$$

$$\frac{\partial \log l_i}{\partial \eta_i} = -\frac{\epsilon_i}{1+\epsilon_i}\left(\frac{\alpha_i}{\bar{\eta}} + \frac{1}{1-\eta_i}\right) \tag{14}$$

$$\frac{\partial \log w_j}{\partial \eta_i} = -\frac{1}{1+\epsilon_j}\left(\frac{\alpha_i}{\bar{\eta}}\right) \tag{15}$$

$$\frac{\partial \log l_j}{\partial \eta_i} = -\frac{\epsilon_j}{1+\epsilon_j}\left(\frac{\alpha_i}{\bar{\eta}}\right) \tag{16}$$

Proof 4. *See Appendix A.3*

Equations (13) and (14) are the direct effect of automation , namely, the change in wages and employment of a given occupation subsequent to the rise of the automation of the same occupation. Equations (15) and (16) represent the cross-occupation spillovers. Indeed wages and employment in occupation j fall as automation in occupation i increases. Two observations can be drawn from the proposition. First, cross-occupation spillovers have a negative sign because automated occupations absorb capital from the other occupations. Second, because larger occupations absorb more capital, cross-occupation spillovers are primarily determined by the size of the automated occupation. Automation increases the capital productivity of the automated occupation. The capital market reallocates capital in accordance with Proposition 1. A lower fraction of capital goes to the other occupations, resulting in lower labor demand.

Proposition 4. *The short run automation effect is characterized by*

$$\mathrm{d}\ln w_i = -\Omega_i \left(\Theta_{SR} + \frac{1}{1-\eta_i} \mathrm{d}\eta_i \right) \tag{17}$$

$$l_i = \frac{1}{\phi\tau} \ln\left(\frac{w_i}{\tau}\right) \tag{18}$$

where $\Omega_i = \frac{\Lambda_0(\phi g_i K)}{1+\Lambda_0(\phi g_i K)}$*, and* $\Theta_{SR} = \sum_j \frac{\alpha_j}{\bar{\eta}} \mathrm{d}\eta_j$

Equation (17) is the wage decomposition equation. Cross-occupation spillovers are distinguished by the factor Θ_{SR}. Indeed, it reflects the fact that increased automation of one occupation affects all occupations. The size of occupations experiencing increased automation magnifies the spillover effect. Indeed, when a large-scale occupation is automated, the latter absorbs an inordinate amount of capital from other occupations, resulting in a significant decline in employment and wages. Equation (18) demonstrates that the wage and employment decompositions are similar. In occupations where supply is inelastic, automation's effect is reflected relatively more on the wage rate.

2.2.2. Long Run Effect of Automation

In contrast to the short run, long run automation accounts for changes in the capital stock. The economy shifts from one steady state to another with a higher capital stock as a result of automation. Indeed, the total derivative of the stock of capital is given by the following.

$$\frac{\mathrm{d}K}{K} = \sum_i \frac{\alpha_i}{\bar{\eta}} \left(\Pi + \rho \Pi_i\right) \mathrm{d}\eta_i \tag{19}$$

where $\Pi_j = \frac{\zeta_i \left(1 - \frac{\phi}{\Lambda_0(\phi g_i K)+1}\right)}{\sum_i \zeta_i \frac{\phi g_i}{\Lambda_0(\phi g_i K)(\Lambda_0(\phi g_i K)+1)}}$, and $\Pi = \sum_i \Pi_i g_i$. Equation (19) is derived from the capital stock equation in Proposition 2. It depicts the rate of change in the total capital stock as a result of increased automation at the steady state. Because the quantities Π_j are positive ($\phi < 1$), the capital variation rate is positive.

Lemma 1. *Automation increases the total stock of capital*

Automation increases the total capital stock. This is made possible by increased productivity and higher returns on investment. The following proposition provides the long-term effect of automation after accounting for changes in the capital stock.

Proposition 5. *Long run automation effect is characterized by*

$$\mathrm{d}\ln w_i = -\Omega_i \left(\Theta_{LR} + \frac{1}{1-\eta_i} \mathrm{d}\eta_i\right)$$

where $\Theta_{LR} = \sum_j \alpha_j \frac{1-\Pi-\rho\Pi_j}{\bar{\eta}} \mathrm{d}\eta_j$

The quantity Θ_{LR} represents the long-term cross-occupation spillovers. Indeed, it shows that a change in automation in a given occupation affects all occupations. The long run spillover effect is mitigated because of the increase in the stock of capital.

$$\Theta_{LR} < \Theta_{SR}$$

In the case where Θ_{LR} is positive, the long run spillover effect is negative as well as the short run, but the magnitude is smaller. if Θ_{LR} is negative, then the long rung spillover effect is positive. In that case, the increase in capital cancels out the negative short-run spillover effect.

2.3. Transition Between States

The preceding analysis is a comparison of steady states. As a result of increased automation, the economy shifts from one steady state to another with more capital. The transition of the economy between the two steady states subsequent to a change in automation is represented by the following equations.

$$\frac{\dot{K}(t)}{K(t)} = \sum_i \zeta_i \left(1 - \frac{\phi}{\Lambda_0(\phi\lambda(t)g_i(t)K(t))}\right) g_i(t) + R(t) \tag{20}$$

$$\frac{\dot{\mu}(t)}{\mu(t)} = \rho - R(t) \tag{21}$$

$$R(t) = A\bar{\eta} \prod \left(\frac{\Lambda_0(\phi\lambda(t)g_i(t)K(t))}{\phi\tau K(t)}\right)^{\alpha_i(1-\eta_i)} \tag{22}$$

$$\lambda(t) = \mu^{\frac{1}{\theta}}(t)$$

$$g_i(t) = \frac{\alpha_i(1-\eta_i)}{\bar{\eta}} R(t)$$

Equation (20) represents capital motion as derived from household budget constraints. Equation (21) is the Euler equation. It represents the optimal dynamic behavior of the household. The interest rate in equation (22) is equal to the marginal production of capital.

Prior to the increase in automation, the interest rate equals the discount rate because the economy is in a steady state. Subsequent to automation, the interest rate rises. The increase in the interest rate is caused by the fact that automation increases capital productivity and thus capital demand. However, because the capital supply is inelastic in the short run, households are unable to increase their capital supply. The rise in the interest rates that follows the rise in automation provides an incentive for households to increase their savings. The process results in a gradual increase in the capital stock. However, as the capital supply expands, the interest rate falls, slowing down the expansion of the capital supply. When the interest rate falls to the steady-state value, the capital accumulation comes to an end and the economy reaches a new steady state. In the new steady state the level of capital is higher.

Next, I study the distribution of wages and employment along the transition path.

$$w_i(t) \geq w_j(t) \iff \alpha_i (1 - \eta_i) \geq \alpha_j (1 - \eta_j), \forall i, j \tag{23}$$

Similarly,

$$l_i(t) \geq l_j(t) \iff \alpha_i (1 - \eta_i) \geq \alpha_j (1 - \eta_j), \forall i, j \tag{24}$$

The distributions of wages and employment are determined by the labor share of each occupation (the labor share is the product of the occupation share in the aggregate economy and the labor share within the occupation). In other words, employment and wages are higher in occupations with higher labor share.

$$\frac{\partial^2 w_i(t)}{\partial g_i(t) \partial K(t)} = \frac{\phi\tau \left(\Lambda_0 \left(\phi g_i(t) K(t)\right)^2 + \Lambda_0 \left(\phi g_i(t) K(t)\right) + 1\right)}{\left(\Lambda_0 \left(\phi g_i(t) K(t)\right) + 1\right)^3} > 0, \forall i \in \{1, 2, ..., n\} \tag{25}$$

$$\frac{\partial^2 l_i(t)}{\partial g_i(t) \partial K(t)} = \frac{\Lambda_0 \left(\phi g_i(t) K(t)\right)}{\tau\phi g_i(t) K(t) (\Lambda_0 \left(\phi g_i(t) K(t)\right) + 1)^3} > 0, \forall i \in \{1, 2, ..., n\} \tag{26}$$

Equations (25) and (26) show that the dispersion of employment and wages increases along the transition path. Indeed, employment and wage growth are faster in occupations with higher employment and wages.

3. Quantitative Analysis

In this section, I calibrate the model parameters to the 2010 US economy, then I compute the change in wages and employment due to automation between 1980 and 2010 and decompose this change into the various effects of automation. I also calculate the increase in occupational labor income inequality as a result of automation.

3.1. Data

The analysis is conducted by using two types of data. The first is IPUMS files of American Community Survey data on wages and employment from the 1980 and 2010 censuses. Workers in the sample are civilians aged from 16 to 64. Employment is calculated using full-time full-year equivalent workers. The second type is data on occupations tasks input of Autor and Dorn (2013).

Following Autor (2015), I classify occupational tasks into three categories: manual, routine, and abstract. Abstract tasks are difficult to computerize because they necessitate creativity, intuition, and other cognitive abilities. Manual tasks necessitate situational awareness, visual and language recognition, and face-to-face interaction. Finally, routine tasks are explicit and codifiable tasks (Autor (2015)). The routine share of an occupation (RS_i) is the proportion of routine tasks within an occupation i and is calculated as follows.

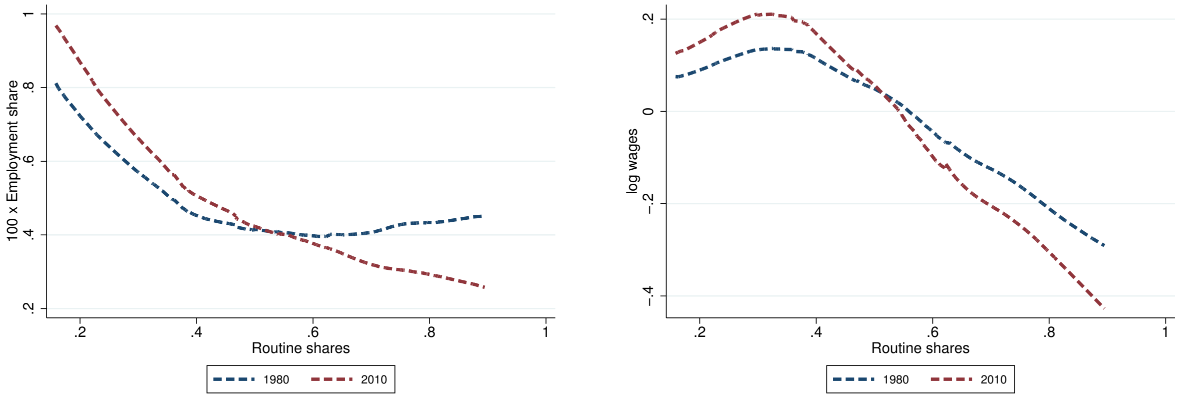
$$RS_i = \frac{T_i^R}{T_i^R + T_i^M + T_i^A} \tag{27}$$

where T_i^R, T_i^M, and T_i^A represent routine, manual, and abstract input of occupation i, respectively.

Figure 1 depicts the employment share and wages with respect to RS. The RS spectrum, represented in the x axis, ranges from the least routine-intensive (i.e. the manual occupations such as services, the abstract occupations such as professionals technicians managers etc.) at the bottom to the most routine-intensive at the top. Routine-intensive occupations are those that have a high RS and are easily automatable. The employment share decreases with the RS, as shown in the left panel of Figure 1. Furthermore, the shares of routine-intensive occupations decrease over time, while the shares of non-routine occupations increase. The use of robotics and artificial intelligence is increasing exponentially, and it is taking over more and more jobs of low-skilled workers, who mostly work in routine-intensive occupations.

Similarly, the right panel follows the same pattern as the employment share. Indeed,

Figure 1: Occupational employment share and wages of 1980 and 2010



Note: ACS data in 1980 and 2010. The occupations are represented on the x axis by the level of their routine shares. The routine shares are computed using occupation tasks input from Autor and Dorn (2013) using equation (27). The curves are smoothed with the stata lowess function with a bandwidth of 0.75.

wage levels are falling as RS rises. However, the relationship is positive at the bottom of the RS spectrum. This is because the RS of manual occupations is lower than the RS of abstract occupations, but the wage level for abstract occupations is higher. However, in general, the relationship is negative because automation has a negative impact on wages.

3.2. Calibration

I assume the following linear relationship to calibrate the automation parameters:

$$\eta_i = \iota RS_i$$

where ι represents the automation intensity. The assumption is that automation is proportional to RS. ι is calibrated to target the aggregate labor share. Indeed, as shown in Corollary 1 the aggregate automation represents the capital share in the model. Therefore the aggregate labor share of the model is $1 - \iota \sum_{i=1}^{n} \alpha_i RS_i$. The parameters α_i are expressed

Table 1: Calibration summary

	Description	Value	Target/Identification	Data	Model
Preferences					
ρ	Discount rate	0.04	Real interest rate=4% (Kaymak and Schott (2019))		
ζ	Weights of households	vector	Employment share		
θ	Inverse IES	3	Moll et al. (2019)		
τ	taste for leisure	0.99	mean of wages normalized to 1		
Technology					
RS	Routine shares	vector	$RS = \frac{T^R}{T^R+T^M+T^A}$ where T^R is routine task input, T^M is manual task input and T^A is abstract task input from Autor and Dorn		
α	Occupations share	vector	$\alpha_i = \frac{\frac{w_i l_i}{1-\eta_i}}{\sum_{i=1}^{n} \frac{w_i l_i}{1-\eta_i}}$		
ι	Automation intensity	0.9	Labor share in 2010	0.57	0.57

in terms of the observables and automation-related parameters in the following equation

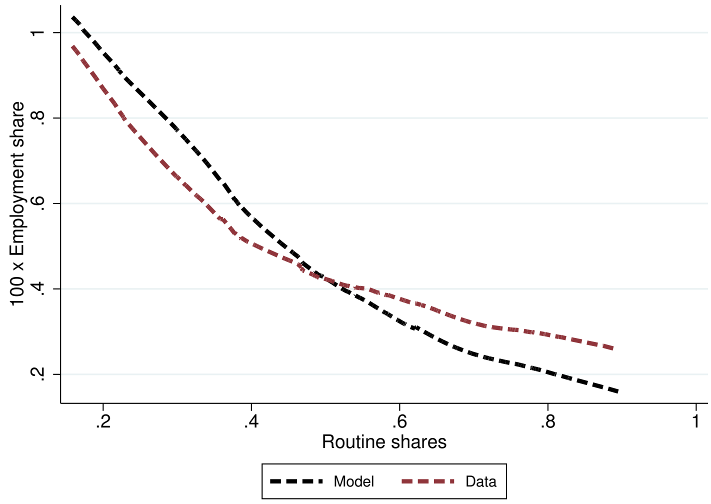
$$\alpha_i = \frac{\frac{w_i l_i}{1-\iota RS_i}}{\sum_{i=1}^{n} \frac{w_i l_i}{1-\iota RS_i}} \tag{28}$$

Proof 5. *See Appendix C*

The discount rate is set to the value of the long-term interest rate as suggested by Proposition 2. The value of the interest rate equals 4% (Kaymak and Schott (2019)). The weights of the households ζ are measured by the employment share. The parameter τ is set so that the average wage equals one. θ, the inverse of the intertemporal elasticity of substitution is set to 3 (Moll et al. (2019)). Table 1 contains the summary of the calibration results.

Figure 2 compares the employment shares in the model with the data. Although the employment share is not targeted, the model-to-data fit is satisfactory. Indeed, the model reproduces the main pattern of the employment share.

Figure 2: Model vs Data employment share



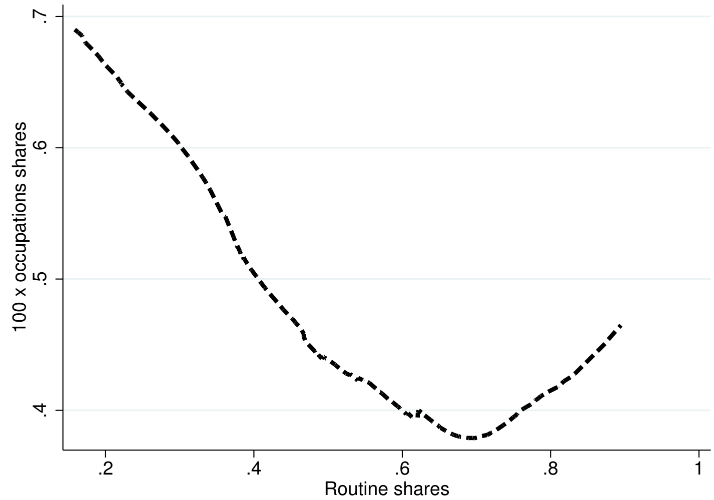
Note: The employment data is smoothed with the stata lowess function and a bandwith of 0.75.

As mentioned in the previous section, increased automation of a large occupation implies a larger spillover effect than increased automation of a smaller occupation, all else being equal. Figure 3 depicts the size of the occupations with respect to RS (i.e. the size of the occupations are represented by the occupations shares α_i). The occupations shares decrease in the first two tertiles of the RS distribution, but increase in the last tertile. It implies that the most routine-intensive occupations have relatively significant size in the economy. When this is combined with the fact that routine-intensive occupations are experiencing the fastest increase in automation, the magnitude of the spillovers is significant.

3.3. Counterfactual Analysis

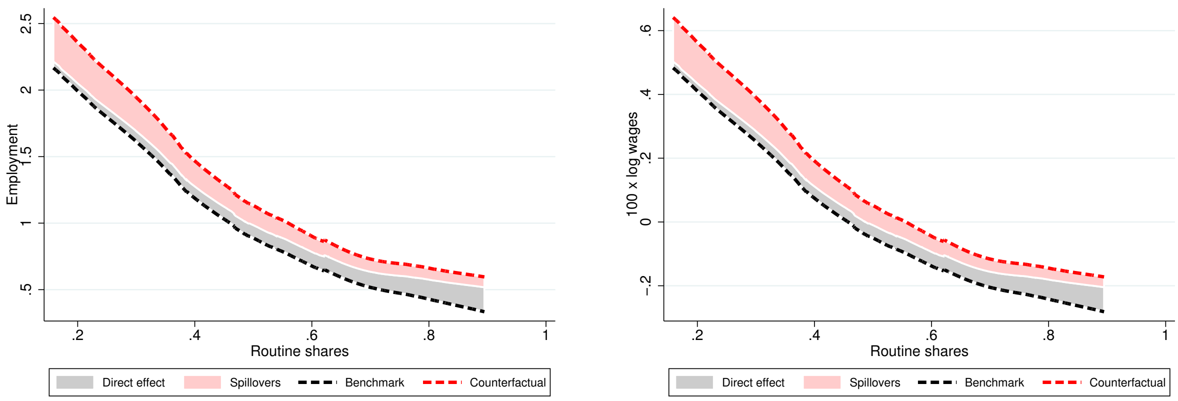
In this subsection, I simulate a counterfactual economy that has a lower automation intensity. The calibrated model presented in the previous section serves as the benchmark. The counterfactual is the calibrated model as the benchmark, except for the automation intensity. The counterfactual economy's automation intensity is set to match the labor share of 1980. The labor share in 1980 was 0.64 (Bureau of Labor Statistics), and the associated automation intensity was 0.78.

Figure 3: Occupations shares



Between 1980 and 2010, the automation intensity has risen from 0.78 to 0.9. The implication of this change in terms of employment and wages is depicted in Figure 4. The left

Figure 4: Decomposition of change in wages and employment



Note: The red line represents the level of automation in 1980, while the black line represents the level of automation in 2010.

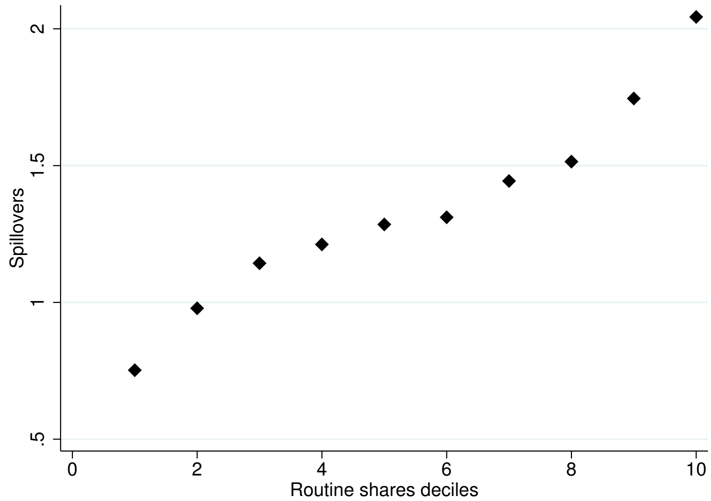
panel depicts the employment per RS, while the right panel depicts the log wage per RS. The red line represents the counterfactual economy and the black line the benchmark economy. The shift from the red line to the black line represents the reduction in employment and wages due to automation between 1980 and 2010. Automation has decreased the average labor income by 27% in this period of time. It implies that automation machines are taking over human labor. This is due to two major reasons. The first reason is that automation can shorten the time it takes to complete a task, compared to humans. The second reason is that automation is a less expensive option than hiring humans for specific tasks.

The decrease in employment and wages is divided into two effects: the direct effect and the cross-occupation spillovers. The grey area represents the direct effect, and the formula, as mentioned in Propositions 4 and 5 , is $\frac{\Omega_i}{1-\eta_i}\mathrm{d}\eta_i$. The magnitude of the direct effect is positively correlated with the RS.

The red area represents the shift in employment and wages caused by cross-occupation spillovers. As shown in Propositions 4 and 5, the magnitude of the cross-occupation spillover is $\Omega_i\Theta_{SR}$ in the short run, and $\Omega_i\Theta_{LR}$ in the long run. Unlike the direct effect, the magnitude of the spillovers is negatively correlated to RS. This is because Ω_i decreases with RS. Ω_i measures the marginal effect of labor productivity change relative to capital productivity on employment and wages. Cross-occupation spillovers account for 62% of the overall change in wages and employment. In Appendix D, I calculate the equilibrium of the benchmark economy and simulate the counterfactual economy when the production function in (3) is a CES rather than a Cobb Douglas production function. Although the magnitude of the spillover is positively related to the substitution elasticity, the main idea prevails in the sense that automation in routine occupations reduces both employment and wages in non-routine occupations.

Figure 5 depicts the spillovers per RS deciles. The spillover for a given decile is calculated as the percentage change in labor income on the other deciles from 1980 to 2010, as a result of increased automation in the considered decile. The spillovers range from 0.75 to 2.04%. Between 1980 and 2010, the increase in automation in the 10% most routine-intensive occupations reduced average labor income by 2.04% in the 90% least routine-intensive occupations. Similarly, automation of the 10% least routine-intensive occupations has reduced the average labor income of the 90% of the most routine-intensive occupations by 0.75%. The spillovers from routine-intensive occupations are the most significant in terms

Figure 5: Spillovers per RS deciles



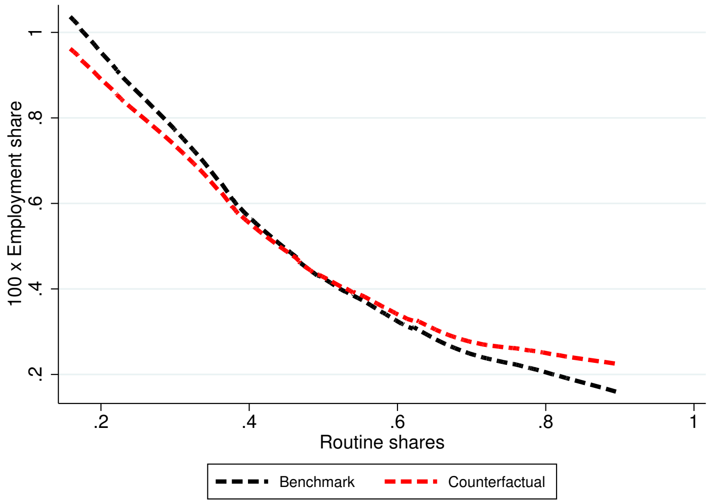
Note: The spillover for a given decile is calculated as the percentage change in labor income on the other deciles from 1980 to 2010, as a result of increased automation in the decile under consideration.

of magnitude. As a result, ongoing computerization and robotization in routine occupations significantly reduces wages and employment in non-routine occupations.

It is worth noting that non-routine occupations have a high dispersion of labor income because they include both abstract occupations like managers and professionals and manual occupations like cleaning services. The latter have lower labor income. Cross-occupation spillovers imply that, while automation is slow in these occupations, automation in routine occupations reduces their labor income.

Automation does not reduce employment uniformly across occupations. The decline is more pronounced in routine-intensive occupations. Thus, automation explains a portion of the decline in employment share in routine-intensive occupations relative to non routine occupations, as shown by the data in Figure 1. Indeed, Figure 6 depicts the occupations employment shares in the benchmark economy (black line) and the counterfactual economy (red line). The gradient of the employment share increases in absolute value with automation. This demonstrates that the intensity of automation reduces the employment share in routine-intensive occupations relative to non-routine occupations. This finding supports the

Figure 6: Employment share and Automation



notion that automation is a skill-biased technological change, in the sens that demand for skilled labor rises relative to demand for less-skilled labor.

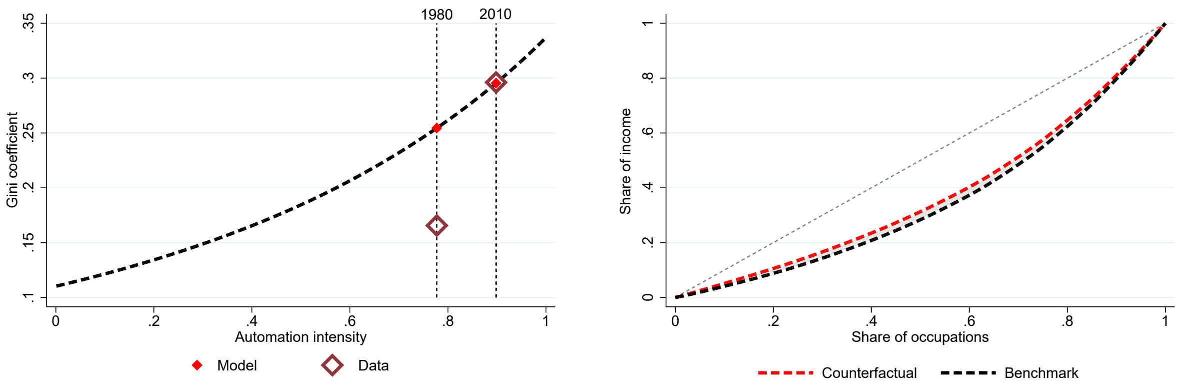
It is important however to note that, while automation will eliminate some jobs, it will also create new ones. Acemoglu and Restrepo (2018b) have documented the emergence of new job titles as a result of automation's ability to generate new jobs.

3.4. Automation and Inequality of Labor Income

The right panel in Figure 7 depicts the Lorenz curves of the benchmark and the counterfactual economies. The benchmark that corresponds to a higher level of automation has the greater level of labor income inequality in comparison to the counterfactual economy.

Figure 7's left panel depicts the Gini coefficient of the labor income level with respect to automation intensity. The value of 0.78 corresponds to the automation intensity in 1980, and the value of 0.9 corresponds to the automation intensity in 2010. The red diamonds represent the Gini coefficients computed in the model in 1980 and 2010, and the maroon diamonds are the corresponding values in the data. The graph shows that automation increases the labor income inequality between occupations exponentially. Automation has

Figure 7: Automation and labor income inequality



Note: The left panel shows the Gini coefficient with respect to automation intensity. The vertical lines represent the automation intensity of 1980 and 2010. The right panel shows the 1980 Lorenz curve (in red) and the 2010 Lorenz curve (the black line).

contributed to the increase in occupational labor income inequality between 1980 and 2010 by 30.3% .

The distributional effect of automation, according to Proposition 3, does not depend on the utility function of the household. Only the contribution to income from wages and employment is affected by the choice of the utility function.

It is worth noting that spillovers exacerbate the fall in average labor income attributable to automation while reducing the impact of automation on inequality. In other words, without spillovers, the gradient of the 2010 curve in Figure 1 would have been larger. Indeed, the capital flow reduces labor income in non automated occupations while moderating the fall in labor income in automated occupations.

4. Conclusion

Although automation is increasing faster in routine occupations than in abstract and manual occupations, the consequences go beyond a reduction in employment and wages in routine occupations only. Indeed, capital mobility across occupations has a significant impact on how labor demand is affected in non-routine occupations when automation increases in routine occupations.

When the automation of a given occupation increases, more capital is allocated to it at the expense of other occupations. As a result, labor demand in these occupations declines, so do wages and employment. The implication is that increased automation in routine occupations affects not only the wages and employment of workers in these occupations, but also workers in non-routine occupations.

I find that between 1980 and 2010 the average labor income has decreased by 27% as a result of automation. Cross-occupation spillovers account for 62% of this drop. The increase in automation in the 10% most routine-intensive occupations reduced average labor income in the 90% least routine-intensive occupations by 2.04% between this time period.

Automation has also contributed to the rise of inequality in the United States. Indeed, between 1980 and 2010, automation contributed to a 30.3% increase in occupational labor income inequality.

The paper also shows that automation increases the stock of capital. This increase is the result of a temporary rise in the interest rate caused by automation. The temporary rise in interest rates is due to the fact that short-term capital supply is inelastic, whereas long-term capital supply is completely flexible. Automation raises the stock of capital, causing the economy to transition from one steady state to another with a higher stock of capital. The increase in the stock of capital mitigates the long-term effect of automation.

It is worth noting that automation not only eliminates jobs, but also creates new ones. Workers who are interested in technology and innovation will have more opportunities in the future job market.

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Appendices

Appendix A. Short Run Comparative Statics

Appendix A.1. Proof of Proposition 1

The following lemma is required to prove Proposition 1.

Lemma 2.

$$\frac{K}{l_i} = \frac{\bar{\eta}}{\alpha_i\left(1-\eta_i\right)}\omega_i$$

where $\omega_i = \frac{w_i}{R}$ *the relative cost of labor.*

Appendix A.2. Proof of Lemma 2

We must first determine the factors demand. Let p_i denote the cost of producing one unit of occupation i, which will be referred to as the price of occupation i. Given the price distribution, producing Y unit of aggregate output necessitates the demand for occupation given by

$$y_k = \prod_{i=1}^{n}\left(\frac{\alpha_k}{\alpha_i}\frac{p_i}{p_k}\right)^{\alpha_i} Y \tag{A.1}$$

The demands for factors are given by the following:

$$k_i = \left(\frac{w_i}{R}\frac{\eta_i}{1-\eta_i}\right)^{1-\eta_i} y_i \tag{A.2}$$

$$l_i = \left(\frac{R}{w_i}\frac{1-\eta_i}{\eta_i}\right)^{\eta_i} y_i \tag{A.3}$$

Next, I use the capital market clearing condition. Because capital can move between occupations, the market clears when total capital supply equals total capital demand.

$$K = \sum_{i=1}^{n}\left(\frac{w_i}{R}\frac{\eta_i}{1-\eta_i}\right)^{1-\eta_i} y_i \tag{A.4}$$

$$\frac{K}{l_i} = \frac{\sum_{j=1}^{n} \left(\frac{w_j}{R} \frac{\eta_j}{1-\eta_j}\right)^{1-\eta_j} y_j}{\left(\frac{R}{w_i} \frac{1-\eta_i}{\eta_i}\right)^{\eta_i} y_i} \tag{A.5}$$

$$= \left(\frac{w_i}{R} \frac{\eta_i}{1-\eta_i}\right) + \sum_{j=1; j \neq i}^{n} \left(\frac{w_j}{R} \frac{\eta_j}{1-\eta_j}\right)^{1-\eta_j} \left(\frac{w_i}{R} \frac{\eta_i}{1-\eta_i}\right)^{\eta_i} \frac{y_j}{y_i} \tag{A.6}$$

$$= \frac{\bar{\eta}}{\alpha_i (1-\eta_i)} \omega_i \tag{A.7}$$

Let's now prove the proposition.
$\frac{K}{\bar{\eta}} = \frac{l_i}{\alpha_i (1-\eta_i)} \omega_i = \frac{l_i}{\alpha_i} \left(\frac{\omega_i}{1-\eta_i}\right) = \frac{l_i}{\alpha_i} \left(\frac{k_i}{\eta_i l_i}\right) = \frac{k_i}{\alpha_i \eta_i}$
where the first equality comes from Lemma 2. The third equality comes from the factors demand ratio.

Appendix A.3. Proof of Proposition 3

Consider the Lemma 2, in which the LHS labor demand is replaced by the labor supply as a result of market clearing conditions. I then take the log and the derivative with respect to η_i to get the first two equations, and the derivative with respect to η_j to get the last two equations.

Appendix B. Steady State

Appendix B.1. Solution of the household's problem

Let's denote by $\nu(l_i) = \tau l_i$. We can deduce the following from theorem 7.14 in Acemoglu (2009):

$$\mathrm{e}^{-\nu(l_i)} \left(C \mathrm{e}^{-\nu(l_i)}\right)^{-\theta} = \mu(t) \tag{B.1}$$

$$C \nu'(l_i) \mathrm{e}^{-\nu(l_i)} \left(C \mathrm{e}^{-\nu(l_i)}\right)^{-\theta} = w(t) \mu(t) \tag{B.2}$$

$$\rho \mu(t) - \dot{\mu}(t) = R(t) \mu(t) \tag{B.3}$$

$$\text{Equation (B.3)} \Rightarrow \frac{\dot{\mu}(t)}{\mu(t)} = \rho - R(t)$$

$$\text{Equations (B.1) and } B.2 \Rightarrow \nu'(l_i) = \frac{w}{C} \tag{B.4}$$

$$\Rightarrow \left(\mathrm{e}^{-\nu(l_i)}\right)^{1-\theta} = \left(\frac{w}{\nu'(l_i)}\right)^{\theta} \mu(t) \tag{B.5}$$

$$\Rightarrow \left(\nu'(l_i)\right)^{\theta} \left(\mathrm{e}^{-\nu(l_i)}\right)^{1-\theta} = w^{\theta} \mu(t) \tag{B.6}$$

Appendix B.2. Solution of the producer's problem

I begin by pricing factors at their marginal productivity

$$R(t) = A\bar{\eta}K^{\bar{\eta}-1}\prod_{i=1}^{n} l_i^{\alpha_i(1-\eta_i)} \Rightarrow \prod_{i=1}^{n} l_i^{\alpha_i(1-\eta_i)} = \frac{R(t)}{A\bar{\eta}K^{\bar{\eta}-1}}$$

$$w_i = A\alpha_i(1-\eta_i)K^{\bar{\eta}}\frac{\prod_{i=1}^{n} l_i^{\alpha_i(1-\eta_i)}}{l_i}$$

The above equation yields the following results:

$$w_i = A\alpha_i(1-\eta_i)K^{\bar{\eta}}\frac{R(t)}{A\bar{\eta}K^{\bar{\eta}-1}}\frac{1}{l_i} \Rightarrow l_i^d = \frac{\alpha_i(1-\eta_i)R(t)}{w_i\bar{\eta}}K$$

(B.3) gives the steady state interest rate. The labor clearing condition gives the function of wages and employment for a given stock of capital. The capital stock is determined by the following equation

$$\sum_{i=1}^{n} \zeta_i \dot{a}_i(t) = 0 \tag{B.7}$$

Appendix C. Quantitative Analysis

Proof of equation (28)

The proof uses the aggregate production function in Corollary 1. The marginal product of labor in occupation i equals w_i.

$$\frac{\alpha_i\left(1-\eta_i\right)}{l_i}Y = w_i \Rightarrow \alpha_i Y = \frac{w_i l_i}{1-\eta_i} \tag{C.1}$$

Recall that $\sum \alpha_i = 1$. The expression Y is obtained by adding the sum of both sides.

$$Y = \sum_{i=1}^{n} \frac{w_i l_i}{1-\eta_i} \tag{C.2}$$

Finally, the results are obtained by combining C.1 and C.2.

Appendix D. CES Production Function Case

The purpose of this section is to examine the robustness of the various results in the main paper when a CES production function is considered. The CES has the advantage of allowing us to study the consistency of cross-occupation spillovers for more or less substitutable occupations. The cost is that the CES is not tractable, preventing us from obtaining analytical results. The production function is the following

$$Y = \left(\sum_{i=1}^{n} \alpha_i y_i^{\varrho}\right)^{\frac{1}{\varrho}} \tag{D.1}$$

The substitution parameter is ϱ. If $\varrho = 1$, the occupations are perfectly substitutes. The Cobb-Douglas case is represented by $\varrho = 0$. If $\varrho < 0$, then the occupations are gross complements. The production structure within an occupation remains unchanged from the main paper. The first result is the variant of Proposition 1 for the CES case.

Proposition 6. *Let K denote the total stock of automating capital available to the producer, and the production function is given by D.1. The optimal allocation of capital is such that*

$$k_i = \frac{\tilde{\alpha}_i \eta_i}{\tilde{\eta}} K \tag{D.2}$$

where $\tilde{\alpha} = \left(\frac{\alpha_i}{p_i^{\varrho}}\right)^{\frac{1}{1-\varrho}}$, and $\tilde{\eta} = \sum_{i=1}^{n} \tilde{\alpha}_i \eta_i$.

The definition of p_i is the same as in the main paper. The capital allocation structure is the same with the CES as it is with the Cobb Douglas. The capital is allocated based on the shares of the factors. However, the allocation is also affected by occupation prices and substitution elasticity. When the occupations are substitutable, the producer allocates less capital to the more costly occupations and vice versa. When occupations are complementary, however, he does the opposite. Thus, the magnitude of the spillovers is inversely related to the substitution parameter ϱ. The steady state is characterized by the following system of equations

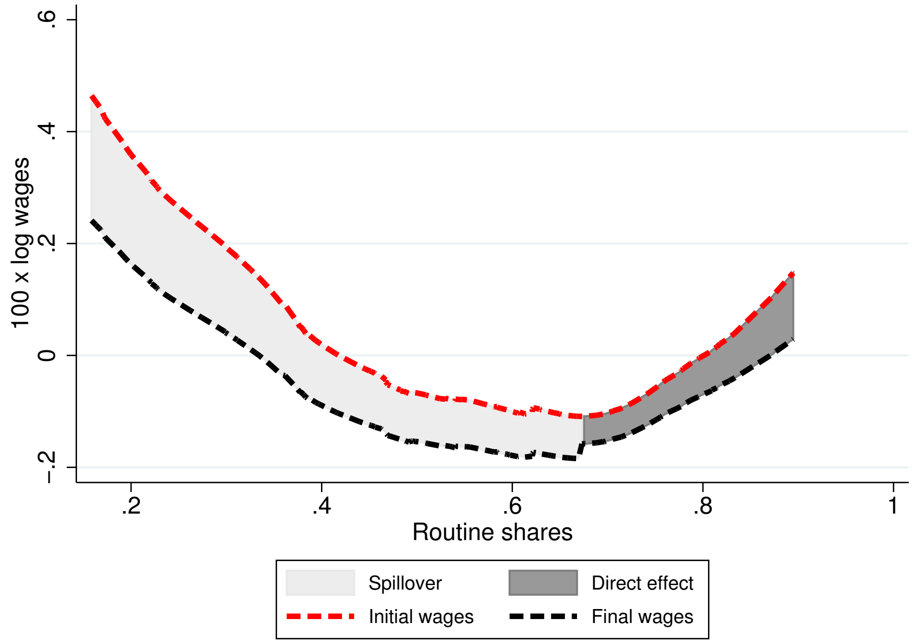
$$\begin{cases} K = \frac{1}{\rho} \sum \zeta_i \left(\frac{w_i}{\tau} - w_i l_i\right) & (1) \\ l_i = \frac{1}{\phi\tau} \ln\left(\frac{w_i}{\tau}\right) & (2) \\ \frac{w_i l_i}{\rho K} = \frac{(1-\eta_i)\beta_i \left(K^{\eta_i} l_i^{1-\eta_i}\right)^{\varrho}}{\sum \eta_i \beta_i \left(K^{\eta_i} l_i^{1-\eta_i}\right)^{\varrho}} & (3) \end{cases}$$

where $s_i = \frac{\tilde{\alpha}_i \eta_i}{\sum \tilde{\alpha}_i \eta_i}$, and $\beta_i = \alpha_i \left(\eta_i^{\eta_i} (1-\eta_i)^{1-\eta_i} s_i^{\eta_i}\right)^{\varrho}$.

The first equation is the equation of capital. It is derived from the budget constraints of households. The second equation represents the labor supply function. The third equation is the labor demand. I use the same parameter values as in the main paper to analyse cross-occupation spillovers with a CES production function. In the figures below, the red line represents the 1980 automation intensity of 0.78. The black line represents an economy in which the automation intensity has increased to 0.9, but only for occupations with RS in the heavy grey area. Automation intensity of occupations in the light grey zone is held constant. So the change in wages in this zone is solely due to spillovers.

Figure D.11 depicts the wage distribution with respect to Frey and Osborne (2013)

Figure D.8: CES case when occupations are gross substitute $\varrho = 0.5$



estimates of the likelihood of occupation automation. As with RS, the gradient is negative.

Figure D.9: Cobb-Douglas case $\varrho = 0$

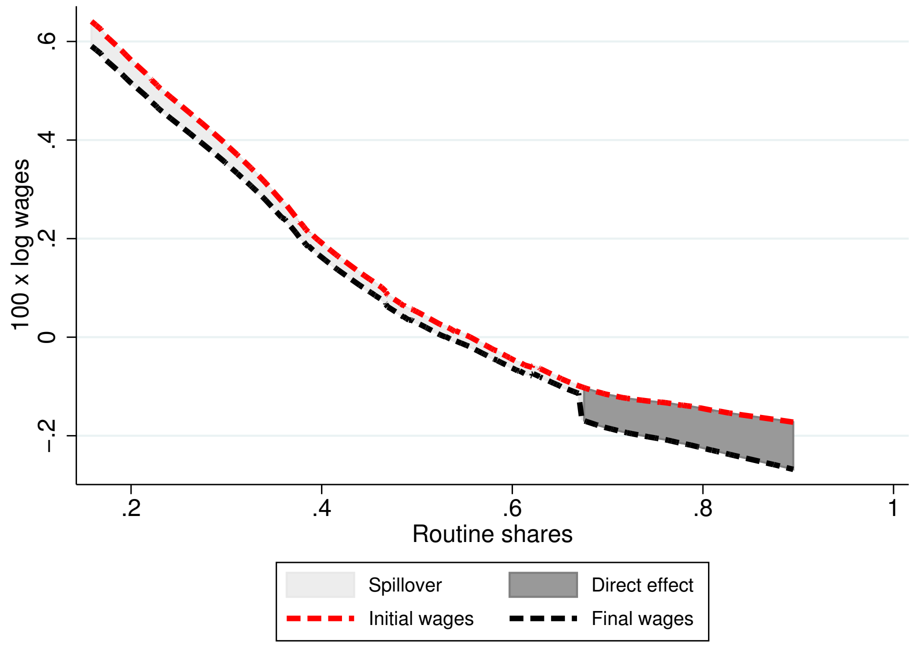


Figure D.10: CES case when occupations are gross complement: $\varrho = -0.5$

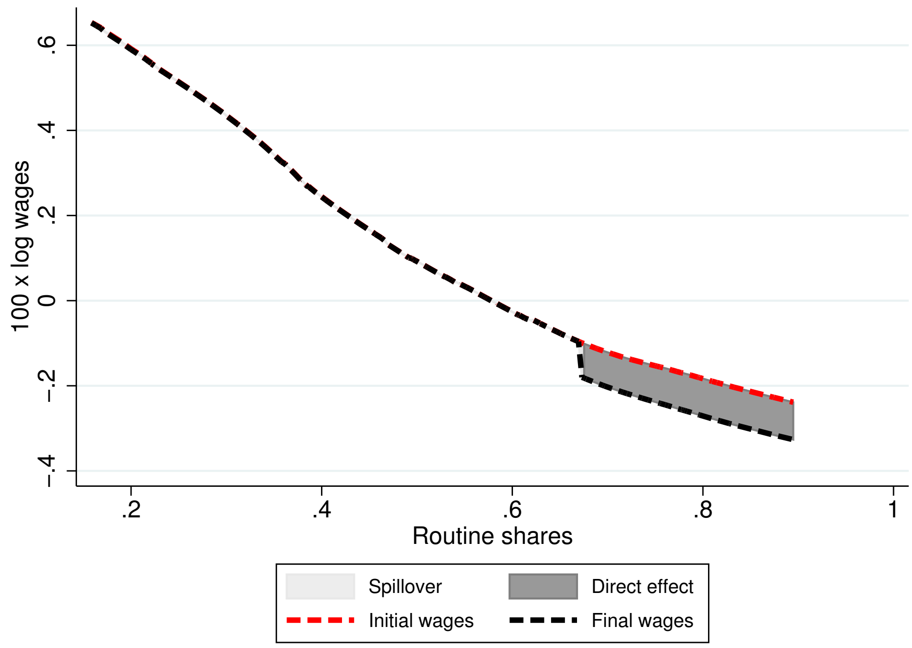
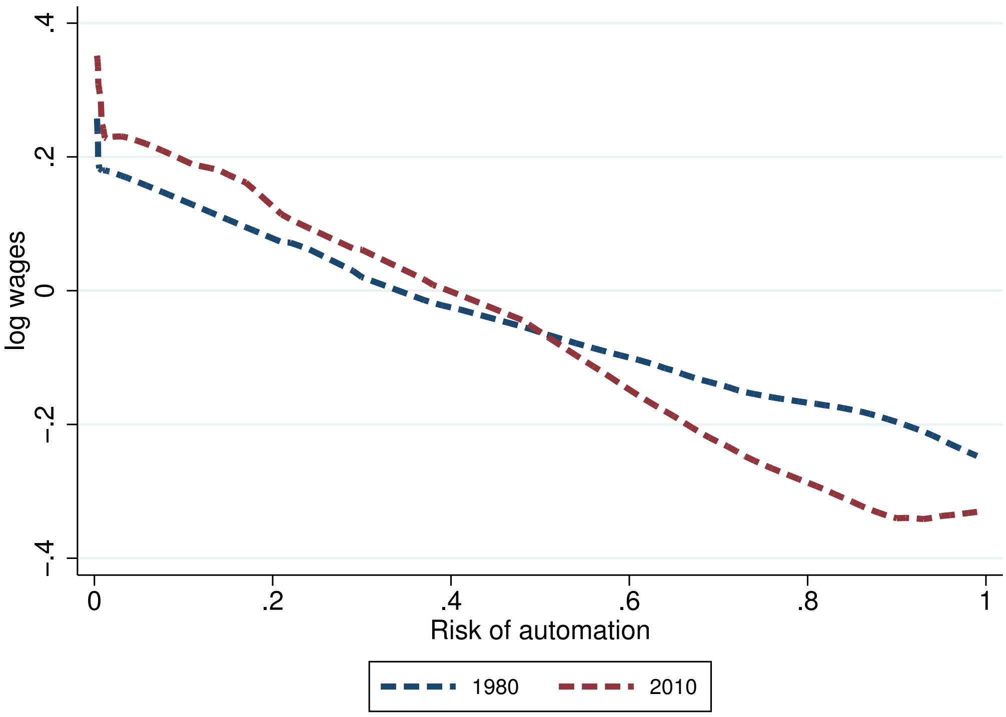


Figure D.11: Occupational wages of 1980 and 2010



Note: The occupations are represented on the x axis by the level of their risk of automation estimated by Frey and Osborne (2013). The curves are smoothed with the stata lowess function with a bandwidth of 0.75.